Surface Impedance Boundary Condition with Circuit Coupling for the 3D Finite Element Modeling of Wireless Power Transfer

Alexis Desmoort[1], Zacharie De Grève[1], Patrick Dular[2], Christophe Geuzaine[2], and Olivier Deblecker[1]

[1]Electrical Power Engineering Unit, University of Mons, Mons B-7000, Belgium

[2]Department of Electrical Engineering and Computer Science, ACE, University of Liège, Liège B-4000, Belgium

alexis.desmoort@umons.ac.be

***Abstract*—A light 3D finite element magnetodynamic a-v model of resonant wireless power transfer coils using 3D surface impedance boundary condition strongly coupled with an external circuit is proposed, reflecting the importance of external circuit elements (notably capacitances) in the resonance phenomena. Preliminary results outline the accuracy of the model.**

***Index Terms*—Finite Element, Inductive Power Transfer**

I. Context and objectives

The 3D finite element modeling of the coils used for resonant Wireless Power Transfer (WPT) as massive conductors is submitted to important computational burden. The skin and proximity effects appearing at the working frequency level (tens of kHz for the transfer of high power) require a high refinement of the conductors' volume mesh, making massive conductors formulations hardly applicable. Here, profit is taken from a 3D Surface Impedance Boundary Condition (SIBC) [1] to avoid the volume mesh inside the conductors and relaxing the computational constraints. Reflecting the important influence of the external circuit on the power transfer (through the resonant effect), a natural circuit coupling is applied leading to a new way to implement SIBC.

II. Methodology

The methodology is based on the extension of [2] to SIBCs. The magnetodynamic **a**-v weak formulation (with **a** the magnetic vector potential and v the electric scalar potential) is obtained from the weak form of the Ampère's equation:

$$\left(\mu^{-1} \text{ curl } \mathbf{a}, \text{curl } \mathbf{a'}\right)_{\Omega} + \langle \mathbf{n} \times \mathbf{h}, \mathbf{a'} \rangle_{\Gamma} + \left(\sigma\left(\partial_t \mathbf{a} + \text{ grad } v\right), \mathbf{a'}\right)_{\Omega_c} = 0 \;\forall\; \mathbf{a'} \in F^1(\Omega) \quad (1)$$

where Ω is the whole domain (of boundary Γ), Ω_c is the massive conductors domain, $F^1(\Omega)$ is a curl-conform function space defined on Ω, **n** is the unit normal vector exterior to Ω, **h** is the magnetic field, σ is the conductivity and μ is the permeability. $(.,.)_\Omega$ and $\langle .,. \rangle_\Gamma$ denote respectively a volume integral in Ω and a surface integral on Γ of the product of their arguments. Massive conductors Ω_c can be extracted from Ω by imposing an SIBC on their boundary (denoted by Γ_c):

$$\mathbf{n} \times \mathbf{h}|_{\Gamma_c} = Z_c^{-1}\; \left(\mathbf{n} \times \left(\partial_t \mathbf{a} + \text{ grad } v\right)\right) \times \mathbf{n}\; |_{\Gamma_c} \quad (2)$$

with Z_c the first order surface impedance of conductors Ω_c:

$$Z_c = (\sigma\; \delta)^{-1}\; (1+j) \text{ with } \delta = \sqrt{2\,(\omega\sigma\mu)^{-1}} \quad (3)$$

where $\omega = 2\pi f$ is the angular frequency and j is the imaginary unit. The coupling of a conductor i with an external circuit is implemented by defining two infinitely close electrodes modeled as a single contour cut γ_j^i in the tube modeling the conductor. Let V_i and I_i be respectively the voltage and the current between and across the electrodes. V_i is strongly determined by defining v as $v = \sum_{i \in \gamma_j} V_i\; v_{0,s}^i$ with $v_{0,s}^i$ a basis function supported here by the surface mesh of Γ_c^i only, equal to 1 on one side of γ_j^i and decreasing linearly to 0 in the first adjacent layer of elements of Γ_c^i. The introduction of the SIBC and of this particular definition of v in (1) leads to the final weak form, whose unknowns are the potential **a** and the global voltage V_i of each conductor, $\forall$ **a'** $\in F^1(\Omega)$:

$$\left(\mu^{-1} \text{ curl } \mathbf{a}\;, \text{curl } \mathbf{a'}\right)_{\Omega\backslash\Omega_c} + \langle \mathbf{n} \times \mathbf{h}, \mathbf{a'} \rangle_{\Gamma\backslash\Gamma_c} + \left\langle Z_c^{-1}\; \mathbf{n} \times \partial_t \mathbf{a}, \mathbf{n} \times \mathbf{a'} \right\rangle_{\Gamma_c} + \sum_{i \in \gamma_j} V_i\; \left\langle Z_c^{-1}\; \mathbf{n} \times \text{ grad } v_{0,s}^i, \mathbf{n} \times \mathbf{a'} \right\rangle_{\Gamma_c^i} = 0. \quad (4)$$

The current I_i is weakly obtained by taking **a'** = grad $v_{0,s}^i$:

$$I_i = \left\langle Z_c^{-1}\; \mathbf{n} \times \partial_t \mathbf{a}, \mathbf{n} \times \text{ grad } v_{0,s}^i \right\rangle_{\Gamma_c^i} + V_i\; \left\langle Z_c^{-1}\; \mathbf{n} \times \text{ grad } v_{0,s}^i, \mathbf{n} \times \text{ grad } v_{0,s}^i \right\rangle_{\Gamma_c^i}. \quad (5)$$

III. Preliminary results

Fig. 1 shows the good accuracy of the method for determining a mutual inductance using circuit relations despite using drastically less mesh elements. In the full paper, the application to a full WPT device in resonant state is targeted.

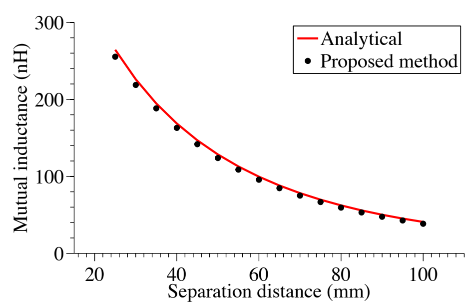

Fig. 1. Mutual inductance extracted from circuit relations for different distances between two identical and aligned 2-turns coils at f = 100 kHz

References

[1] S. Yuferev *et al.*, "Selection of the surface impedance boundary conditions for a given problem," *IEEE Trans. Magnetics*, vol. 35, no. 3, pp. 1486-1489, May 1999.

[2] P. Dular *et al.*, "A general and natural method to define circuit relations associated with magnetic vector potential formulations," *IEEE Trans. Magnetics*, vol. 35, no. 3, pp. 1630-1633, May 1999.